Spectral properties of ultracold Fermi gases

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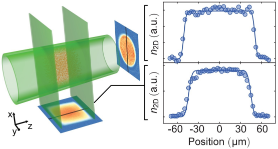
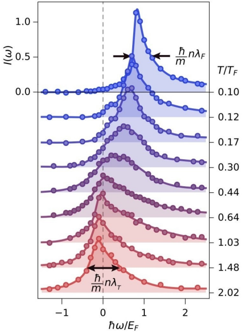
Recent experiments

Homogeneous box potentials

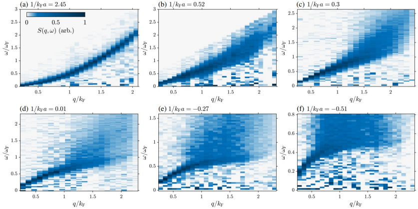


Phys. Rev. Lett. 118, 123401 (2017)

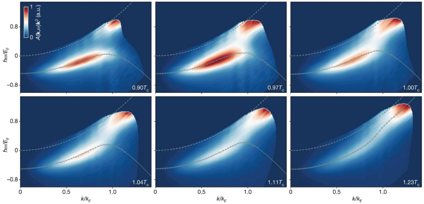


Phys. Rev. Lett. 122, 203402 (2019)

Excitation spectrum and pseudogap



Phys. Rev. Lett. 128, 100401 (2022)



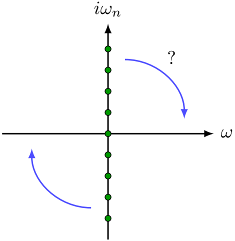
Nature 626, 288–293 (2024)

Why is it so hard?

We want correlation functions at finite temperature.
Standard numerical methods are formulated in imaginary frequencies.

Analytic continuation of numerical data from Matsubara frequencies $i\omega_n$ to real frequencies ω poses an ill-conditioned task with large systematic uncertainties.

Analytic continuation $i\omega_n \to \omega + i0^+$



In this work, we evaluate all quantities directly in real ω.

Model

Spin-balanced two-component Fermi gas in 3D.

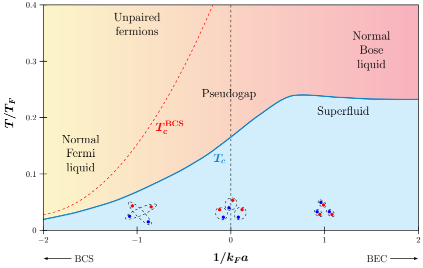
Single-channel model:

$$S[\psi,\phi] = \int_{\tau,\boldsymbol{x}} \Big[\psi_\sigma^* \left(\partial_\tau - \nabla^2 - \mu\right)\psi_\sigma - h\left(\phi^*\psi_\uparrow\psi_\downarrow - \phi\psi_\uparrow^*\psi_\downarrow^*\right) + \nu\phi^*\phi\Big] \,.$$

Feshbach coupling h, detuning ν.

Scattering length: $k_F a = -\frac{h^2}{8\pi\nu}$.

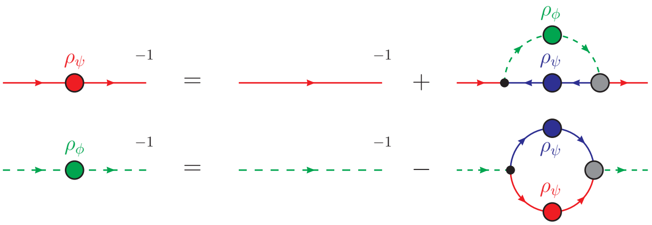
BCS-BEC crossover phase diagram



Hubbard-Stratonovich transformation

Methods

Spectral Dyson-Schwinger equations:



Spectral representation:

$$G(i\omega_n, \boldsymbol{p}) = \int_{-\infty}^{\infty} d\lambda \, \frac{\rho(\lambda, \boldsymbol{p})}{-i\omega_n + \lambda} \, .$$

Spectral function:

$$\rho(\omega, \boldsymbol{p}) = \frac{1}{\pi} \, \mathrm{Im} \, G(\omega + i0^+, \boldsymbol{p}) \, .$$

Analytic continuation

Insert spectral representation in loop integrals:

$$\Sigma(i\omega_n, \boldsymbol{p}) \sim \int_{\boldsymbol{q}} T \sum_{\epsilon_m} G_1(i\epsilon_m, \boldsymbol{q}) G_2(i\epsilon_m - i\omega_n, \boldsymbol{q} - \boldsymbol{p}) \,.$$

Calculate Matsubara sums analytically:

$$\Sigma(i\omega_n, \boldsymbol{p}) \sim \int_{\boldsymbol{q}} \int_{\lambda_1, \lambda_2} \rho_1(\lambda_1, \boldsymbol{q}) \rho_2(\lambda_2, \boldsymbol{q} - \boldsymbol{p}) I(i\omega_n, \lambda_1, \lambda_2) \,.$$

$\longrightarrow$ Analytic continuation $i\omega_n \to \omega + i0^+$ possible!

Define retarded self-energy:

$$\Sigma^R(\omega, \boldsymbol{p}) = \Sigma(\omega + i0^+, \boldsymbol{p}) \,.$$

Evaluation at real frequencies

Imaginary part at real frequencies ω:

$$\boxed{\mathrm{Im}\,\Sigma_\psi^R(\omega,\boldsymbol{p})} \sim \int_{\lambda,\boldsymbol{q}} \boxed{\rho_\phi(\omega+\lambda,\boldsymbol{q})}\,\boxed{\rho_\psi(\lambda,\boldsymbol{q}-\boldsymbol{p})} \times \left[-n_B(\omega+\lambda) - n_F(\lambda)\right] ,$$

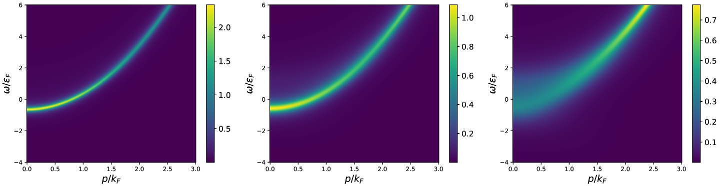
$$\boxed{\mathrm{Im}\,\Pi_\phi^R(\omega,\boldsymbol{q})} \sim \int_{\lambda,\boldsymbol{p}} \boxed{\rho_\psi(\omega-\lambda,\boldsymbol{p})}\,\boxed{\rho_\psi(\lambda,\boldsymbol{q}-\boldsymbol{p})} \times \left[1 - n_F(\omega-\lambda) - n_F(\lambda)\right] ,$$

Real part from Kramers-Kronig relation:

$$\mathrm{Re}\,\Sigma_\psi^R(\omega,\boldsymbol{p}) = \frac{1}{\pi}\,P\int_\lambda \frac{\mathrm{Im}\,\Sigma_\psi^R(\lambda,\boldsymbol{p})}{\lambda-\omega} .$$

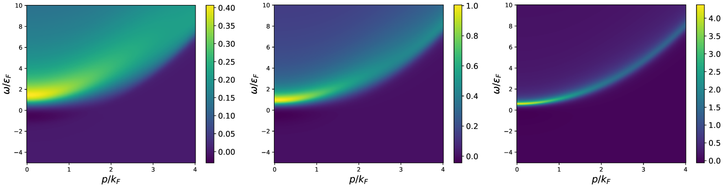
Spectral functions

Fermionic spectral function $\rho_\psi(\omega, \boldsymbol{p})\,\varepsilon_F$ at $T/T_F = 0.56$.



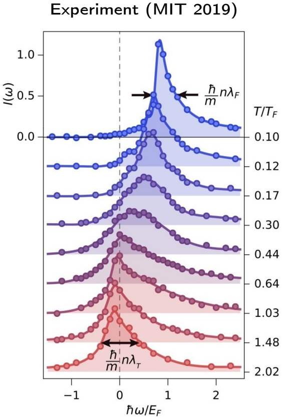
$1/k_F a = -0.5$ $\quad$ $1/k_F a = 0$ $\quad$ $1/k_F a = 0.5$

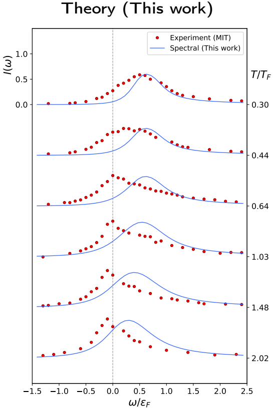
Bosonic dimer spectral function $h^2 \rho_\phi(\omega, \boldsymbol{p})\,\sqrt{\varepsilon_F}/(8\pi)$ at $T/T_F = 0.56$.



$1/k_F a = -0.5$ $\quad$ $1/k_F a = 0$ $\quad$ $1/k_F a = 0.5$

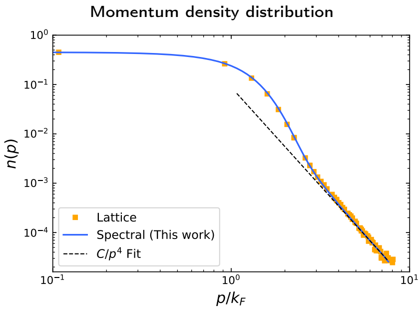
Radio-frequency (Rf) spectra

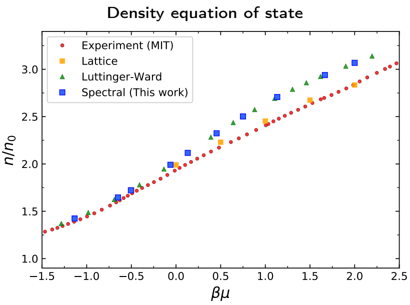




$$I(\omega) = \int_{\boldsymbol{q}} \rho_\psi(\boldsymbol{q}^2 - \omega - \mu, \boldsymbol{q})\, n_F(\boldsymbol{q}^2 - \omega - \mu)$$

Comparison with thermodynamic results





$$n(\boldsymbol{p}) = \int_\lambda \rho_\psi(\lambda, \boldsymbol{p})\, n_F(\lambda)$$

$$n = 2 \int_{\boldsymbol{p}} n(\boldsymbol{p})$$

Outlook

Extensions:

- Full spectral functions in the superfluid phase
- Spin-imbalance and mass-imbalance
- Inclusion of vertex corrections

Applications:

- Bose-Fermi mixtures and superconductors
- Transport properties and shear viscosity
- Sound and heat transport

Thank you for your attention!