Quantum impurities in dipolar Fermi gases

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Abstract

- Impurity physics with dipolar interactions in 3D
- First step towards a dipolar BCS-BEC crossover
- Simultaneous signatures of s- and p-wave pairing

Model

Consider a single dipolar impurity $\hat{d}^\dagger_{\mathbf{k}}$ inside a non-interacting Fermi sea $\hat{c}^\dagger_{\mathbf{k}}$,

$$\hat{H} = \sum_{\mathbf{k}} E_{\mathbf{k}}\, \hat{d}^\dagger_{\mathbf{k}} \hat{d}_{\mathbf{k}} + \sum_{\mathbf{k}} \epsilon_{\mathbf{k}}\, \hat{c}^\dagger_{\mathbf{k}} \hat{c}_{\mathbf{k}} + \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} V(\mathbf{q})\, \hat{c}^\dagger_{\mathbf{k}+\mathbf{q}} \hat{d}^\dagger_{\mathbf{k}'-\mathbf{q}} \hat{d}_{\mathbf{k}'} \hat{c}_{\mathbf{k}}\,,$$

where $E_{\mathbf{k}} = \mathbf{k}^2/(2M)$, $\epsilon_{\mathbf{k}} = \mathbf{k}^2/(2m)$ and $V(\mathbf{q})$ is the Fourier transform of a regularized dipole-dipole potential with short-range cutoff R_0,

$$V(\mathbf{r}) = \frac{d^2}{r^3}\left(1 - 3\cos^2\theta\right)\left(1 - e^{-r^2/R_0^2}\right)^2 .$$

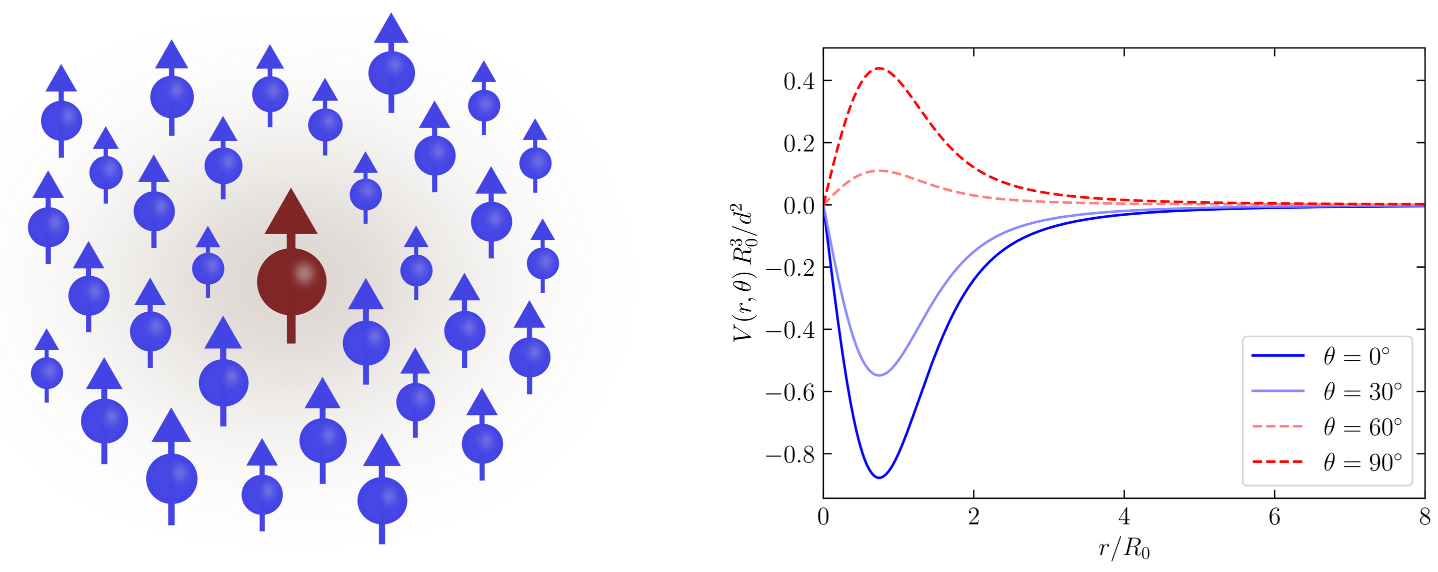

Figure 1. Dipolar polaron in 3D and regularized interaction potential $V(\mathbf{r})$.

Scattering physics

Two-body Schrödinger equation

Using the dipole strength d and reduced mass $\mu = mM/(m+M)$, one can define natural length and energy scales (dipole units),

$$a_d = \mu d^2\,, \quad E_d = 1/(\mu a_d^2)\,.$$

The dimensionless Schrödinger equation,

$$\left[-\frac{\nabla^2}{2} + V(\mathbf{r})\right]\psi(\mathbf{r}) = E\psi(\mathbf{r})\,,$$

is solved by expanding the wave function into partial waves,

$$\psi(r,\theta,\phi) = \frac{1}{r}\sum_{lm} F_{lm}(r) Y_{lm}(\theta,\phi)\,.$$

Scattering lengths

One can define a scattering length a_l for every partial-wave channel,

$$a_l = \lim_{k\to 0} \pi T_{0l}(k)/k\,.$$

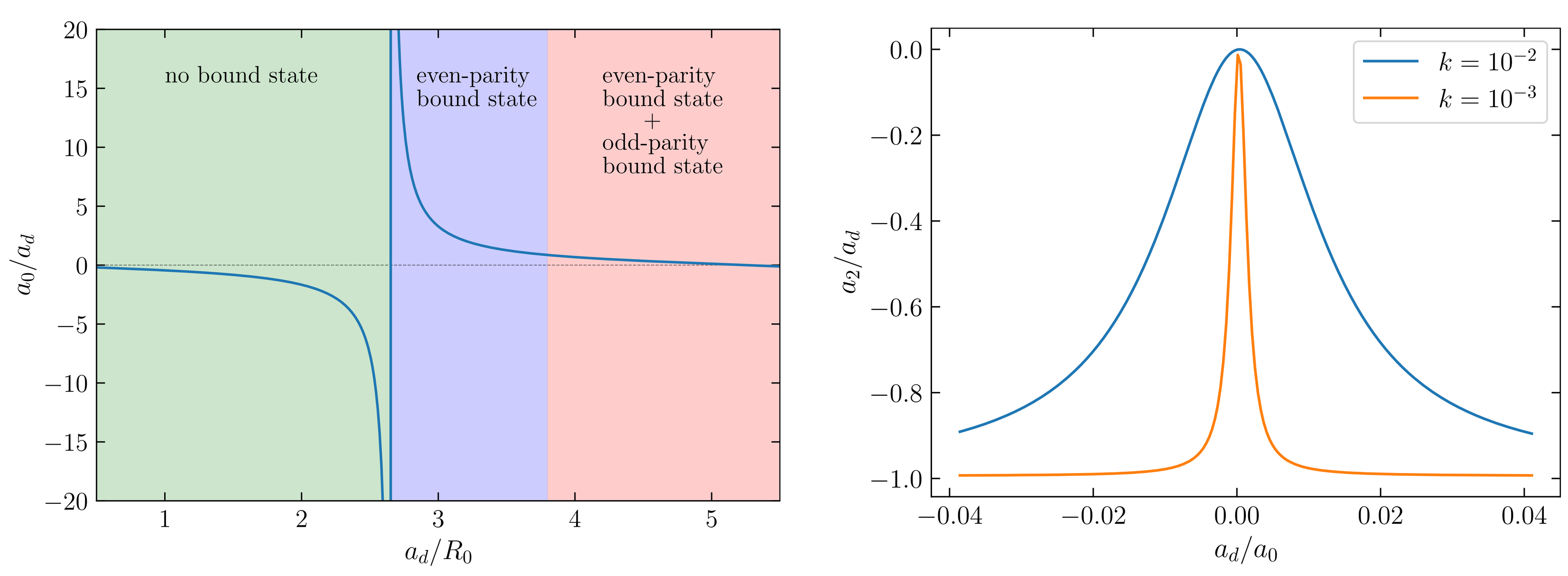

Figure 2. s-wave scattering length a_0 and dipole scattering length a_2.

Effective-range expansion

The low-energy expansion of the s-wave scattering phase shift $\delta_0(k)$ is [1]

$$k^{-1}\tan\delta_0(k) = -a - \frac{76\pi}{525}a_d^2 k - \frac{8a}{15}a_d^2 k^2\ln(k) + O(k^2)\,.$$

Many-body physics

For an infinitely heavy impurity, even and odd angular momenta decouple, and the total many-body spectrum $A(\omega)$ is given by

$$A(\omega) = \frac{1}{\pi}\mathrm{Re}\int_0^\infty dt\, e^{i\omega t} S(t)\,, \quad S(t) = \langle e^{i\hat{H}_0 t} e^{-i\hat{H}t}\rangle\,.$$

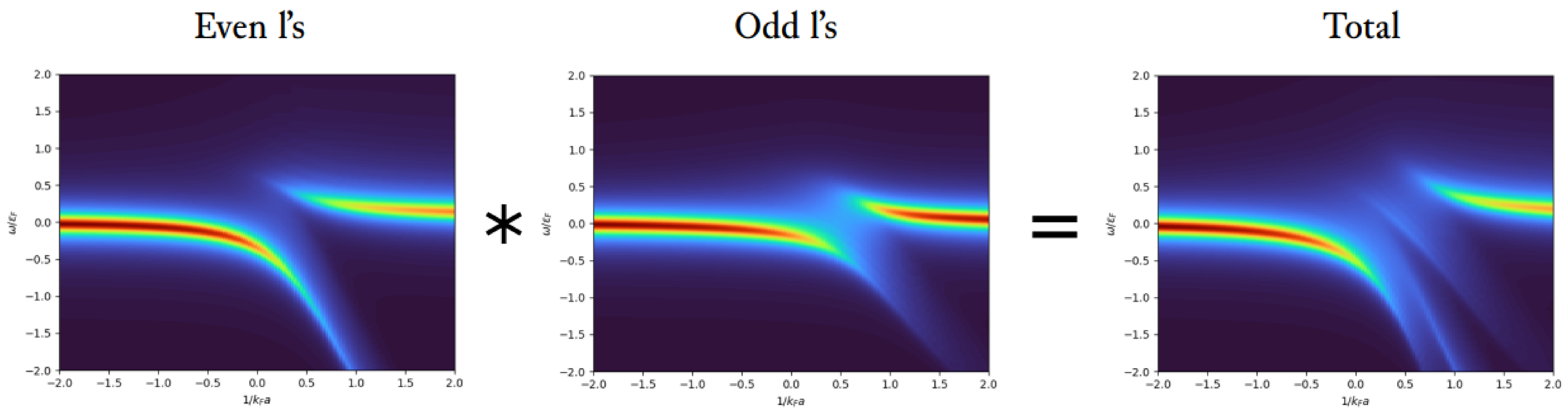

Figure 3. Total spectrum is a convolution of even and odd parity contributions.

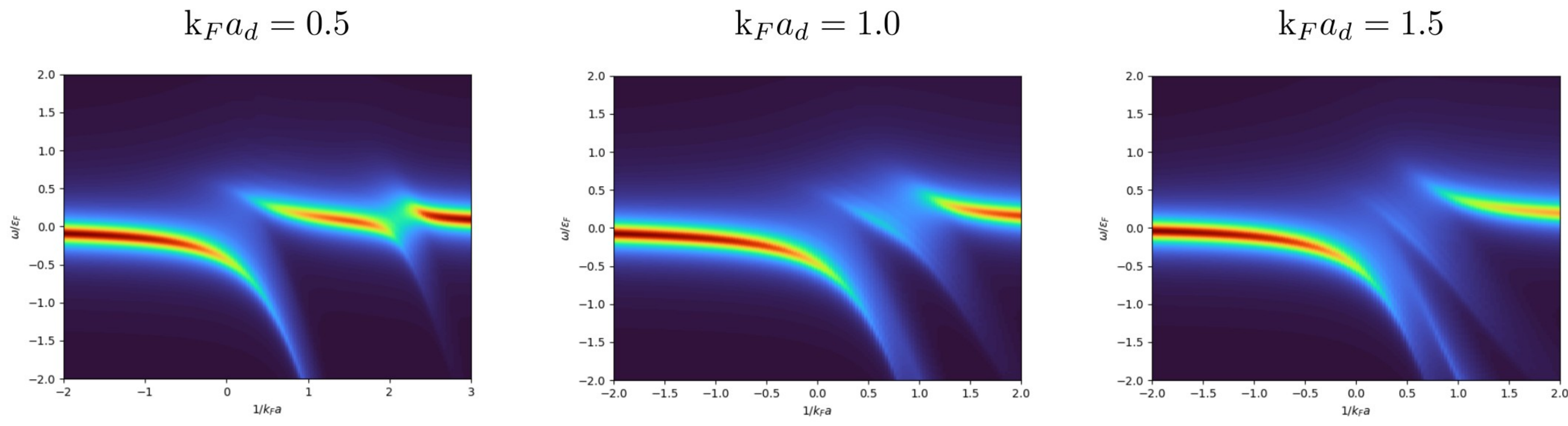

Figure 4. Dipolar polaron spectrum for different dipole lengths $k_F a_d$.

Dipolar polaron energy

The ground state energy is given by generalized Fumi's theorem [2],

$$\Delta E = -\frac{1}{\pi}\int_0^{E_F} dE \sum_{lm}\delta_{lm}(E)\,.$$

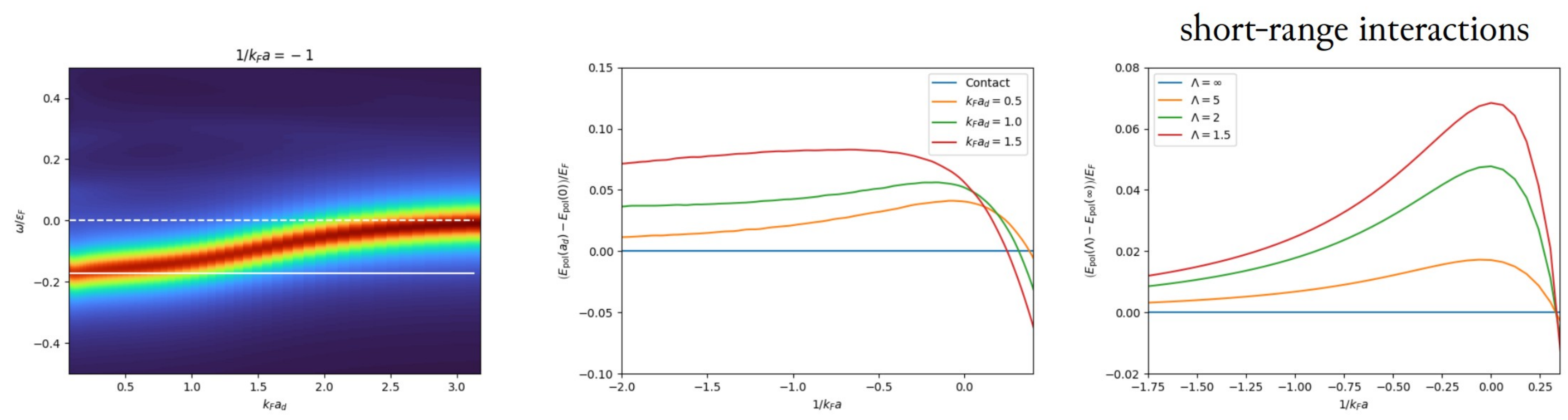

Figure 5. Dipolar polaron energy in comparison to standard short-range interactions.

Orthogonality catastrophe

Quasiparticle weight Z decays as function of fermion number N [2],

$$Z = |\langle \mathrm{FS}|\Psi\rangle|^2 \sim N^{-\alpha}\,, \quad \alpha = \sum_{lm}\left(\frac{\delta_{lm}}{\pi}\right)^2 .$$

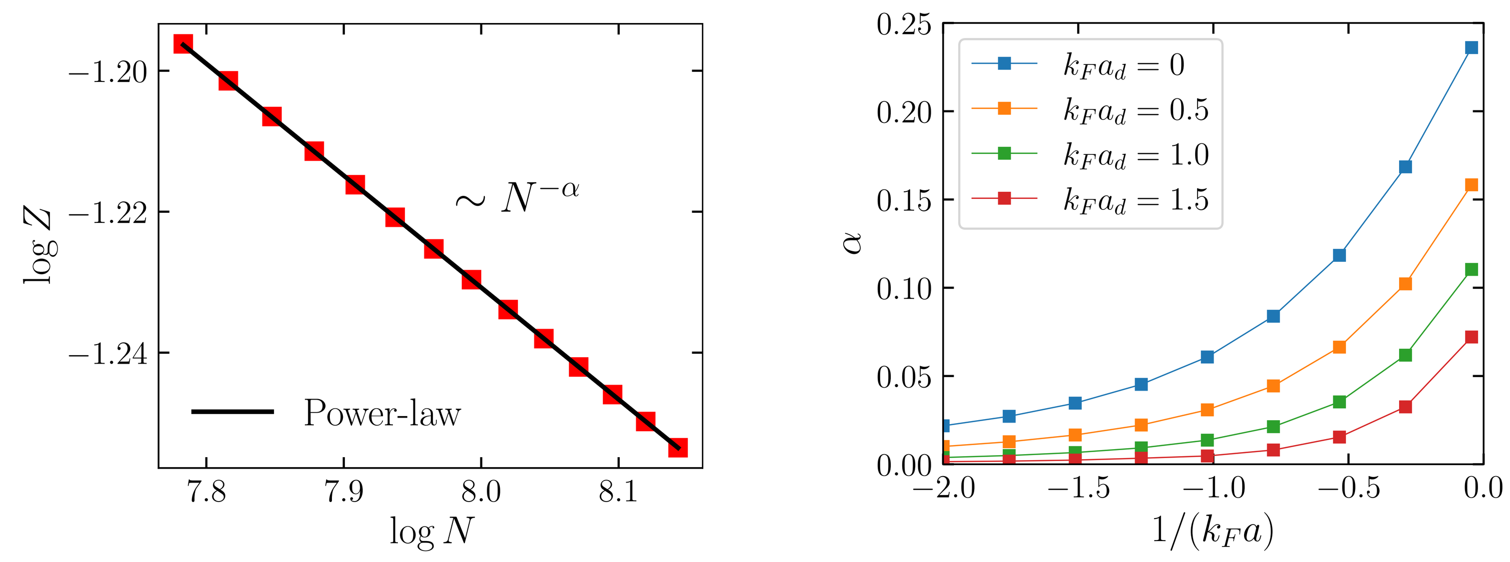

Figure 6. Power-law exponent α of the quasiparticle weight decay $Z \sim N^{-\alpha}$.

References

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[2] M. Combescot and P. Nozières. "Infrared catastrophy and excitons in the X-ray spectra of metals". In: *J. Phys. France* 32 (1971).